Moving Boundaries in 2-D and 3-D TLM Simulations Realized by Recursive Formulas

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Abstract—**In this paper a novel technique for arbitrary boundary positioning in TLM networks will be described. This capability removes the restriction that dimensions of TLM models can only be integer multiples of the mesh parameter and allows superior boundary resolution. Since the position of boundaries can be continuously varied even during a simulation, this feature can model moving boundaries for time domain optimization and phenomena such as the Doppler effect.**

I. Introduction

The accurate modeling of waveguide components, discontinuities and junctions requires a precision in the positioning of boundaries that is identical to, or better than the manufacturing tolerances. In traditional TLM models of electromagnetic structures, boundaries can only be placed either across the nodes or halfway between nodes. Unless all dimensions of the structure are integer multiples of $\Delta l/2$ the mesh parameter would have to be very small indeed, leading to unacceptable computational requirements. Similar considerations apply when curved boundaries with very small radii of curvature must be modeled. It is therefore important to provide for arbitrary positioning of walls. A method for changing the position of boundaries in 2-D TLM through modification of the impulse scattering matrix of boundary nodes has been described already in 1973 by Johns [1] who, at the time, thought that the advantage of this procedure over stepped contour (''Manhattan-style'') modeling was too small to warrant the additional complexity of the algorithm. However, this is not true when analyzing narrowband waveguide components such as filters.

II. Theoretical Background

2.1 Boundary Extension by Reactive Elements

In Johns' concept of arbitrary wall positioning in 2-D TLM [1] a boundary branch which has a length different from $\Delta l/2$ is simply replaced by an equivalent branch of length $\Delta l/2$ having an identical input admittance. This ensures synchronism but requires a different characteristic admittance for the boundary branch and hence, a modification of the impulse scattering matrix of the boundary node.

The method proposed in this paper leaves the impulse scattering matrix intact, but replaces the single boundary reflection coefficient by a recursive reflection algorithm which functions as follows.

Assume that we wish to position a reflecting boundary (electric or magnetic wall) at a distance $\Delta l/2 + l$ from the node as shown in Fig. 1(a)–(b), where l is an arbitrary shift in the boundary position beyond the standard distance $\Delta l/2$ for which the node scattering matrix has to be defined. In fact, this amounts to terminating the regular $\Delta l/2$ long boundary branch in a short- or open-circuited transmission line section with the normalized input reactance

$$z_i = jx_i = \frac{j\omega L}{Z_0} = j \tan \beta l \qquad \text{for an electric wall, and}$$

$$z_i = jx_i = \frac{1}{j\omega C Z_0} = \frac{1}{j \tan \beta l} \qquad \text{for a magnetic wall.} \tag{1}$$

As long as the excess length l is much smaller than the wavelength (or $\beta l << 1$), the inductance or capacitance of the branch extension can be considered independent of frequency, since $\tan \beta l \approx \beta l$, yielding

$$L \approx \frac{Z_0 l}{c} \qquad \text{for an electric wall, and}$$

$$C \approx \frac{l}{c Z_0} \qquad \text{for a magnetic wall} \tag{2}$$

where the propagation velocity on the TLM mesh lines is taken as c.

It is now possible to write the differential equations

$${}_kV_{\text{tot}} = L \frac{\partial_k I_{\text{tot}}}{\partial t} \qquad \text{for an electric wall, and}$$

$${}_kI_{\text{tot}} = C \frac{\partial_k V_{\text{tot}}}{\partial t} \qquad \text{for a magnetic wall} \tag{3}$$

relating voltages and currents at the input of the reactive stubs in terms of incident and reflected impulses, which

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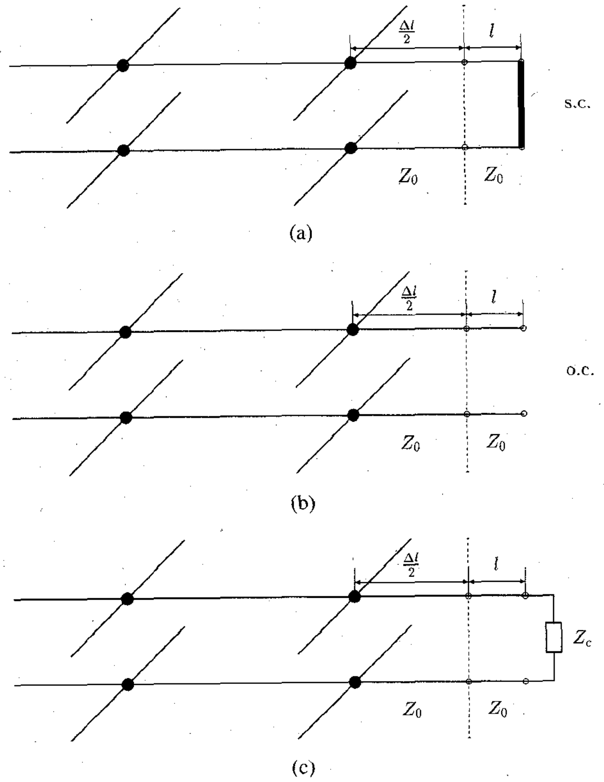

Fig. 1. Extension of (a) short-, (b) open circuit, and (c) arbitrary reflecting boundaries in a TLM mesh.

means that at each computational step we can write

$$_kV_{\text{tot}} = {_kV^r} + {_kV^i}, \quad \text{and}$$

$$_kI_{\text{tot}} = \frac{1}{Z_0}({_kV^r} - {_kV^i}), \tag{4}$$

and to replace these differential equations by difference equations. This results in the following general recursive formula

$$_kV^i = \rho\frac{1-\kappa}{1+\kappa}{_kV^r} + \frac{\kappa}{1+\kappa}(\rho_{k-1}V^r + {_{k-1}V^i}) \tag{5}$$

where $\rho = +1$ for a magnetic wall and $\rho = -1$ for an electric wall. κ is equal to $2l/\Delta l$ in the 3-D TLM case, and $\sqrt{2l/\Delta l}$ in the 2-D TLM case. Equation (5) indicates that the present impulse reflected from the boundary in the reference plane at $\Delta l/2$ depends on the present incident impulse as well as on the previous incident and reflected impulses, which need to be stored. This recursive algorithm amounts to a numerical procedure for integrating the differential equation describing the behavior of the reactive stub in the time domain.

2.2 Stability of the Recursive algorithm

To guaranty an undisturbed simulation the algorithm has to fulfill the requirement of stability. It can be stated in general that negative values for the positioning parameter κ are not allowed, since in that case the algorithm would model negative inductances L and negative capacitances C for electric and magnetic walls, respectively.

In addition to this we have to demand stability for positive values of κ, too. Therefore, we apply the Z-transform to the recursive formula in (5) using the properties

$$Z\{a_kf_1 + b_kf_2\} = aZ\{_kf_1\} + bZ\{_kf_2\} \text{ (Linearity)} \tag{6}$$

and

$$Z\{_{k-n}f\} = z^{-n}Z\{_kf\} \text{ (Time Shifting)} \tag{7}$$

which leads to the equation

$$_kV^i\left(1 - z^{-1}\frac{\kappa}{1+\kappa}\right) = {_kV^r}\rho\left(\frac{1-\kappa}{1+\kappa} + z^{-1}\frac{\kappa}{1+\kappa}\right). \tag{8}$$

Defining the transfer function (Fig. 2) between the incident voltage $_kV^i$ and the reflected voltage $_kV^r$ to be

$$H(z) = \frac{_kV^i}{_kV^r} \tag{9}$$

we get

$$H(z) = \rho\frac{z\frac{1-\kappa}{1+\kappa} + \frac{\kappa}{1+\kappa}}{z - \frac{\kappa}{1+\kappa}} \tag{10}$$

where we can apply stability criteria.

Stability for the case of a time discrete system means that all roots of the denominator of the transfer function $H(z)$ have to be located inside the unit circle of the Gaussian plane. Hence, it results that the algorithm is stable for positive values of the positioning parameter κ. Therefore, the positioning of the moving wall with $\kappa \geq 1$ is possible too, but exhibits a lack of information, since interconnections between the neighboring nodes are missing.

2.3 Boundary Extension by a Combination of a Reactive Element and a Resistor

Assume now a general reflecting wall to be positioned at a distance $\Delta l/2 + l$ from the node as shown in Fig. 1(c), where l is once again an arbitrary shift in the boundary position beyond the standard distance $\Delta l/2$.

Since the termination Z_c can be either larger or smaller than the characteristic impedance Z_0 of the TLM mesh lines, it is necessary to distinguish between two cases. In both cases it is assumed that the excess length is much smaller than the wavelength, and therefore,

$$\beta l << 1 \rightarrow \tan\beta l \approx \beta l \tag{11}$$

and

$$\beta l << 1 \rightarrow \tan^2\beta l \approx 0. \tag{12}$$

Furthermore, let us define the ratio of impedances to be

$$r = \frac{Z_c}{Z_0} \tag{13}$$

and the positioning parameter κ as done earlier.

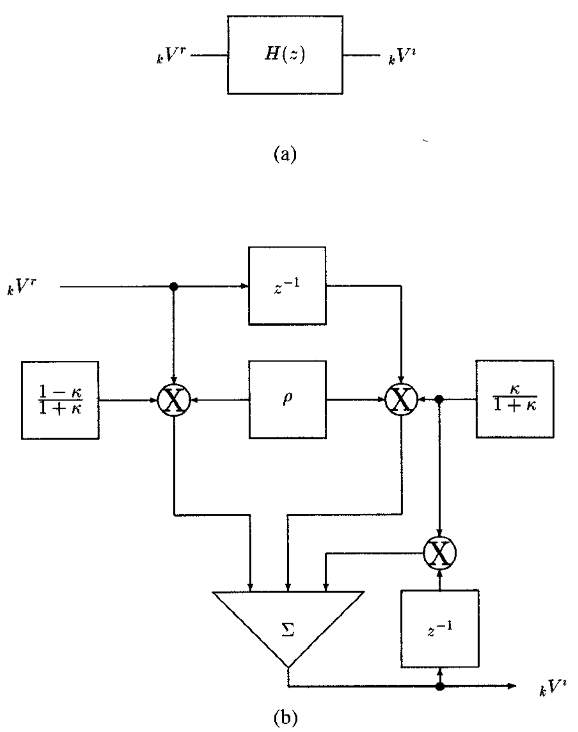

Fig. 2. (a) Definition of the transfer function between the incident and reflected voltages, and (b) the representation of the recursive formula as in (5) by a digital filter.

Moreover, the normalized input impedance z_i of the line connected to the reference plane can be found to be

$$z_i = \frac{r + j \tan \beta l}{1 + jr \tan \beta l} \tag{14}$$

in general.

Case 1: $r < 1$

In this case the behavior of the connected piece of line can approximately be described by a series connection of a resistor R_i and an inductance L_i with

$$R_i \approx rZ_0 \tag{15}$$

and

$$L_i \approx (1 - r^2) \frac{Z_0 l}{c}. \tag{16}$$

Replacing the differential equation

$${}_kV_{\text{tot}} = R_i \, {}_kI_{\text{tot}} + L_i \frac{d \, {}_kI_{\text{tot}}}{dt} \tag{17}$$

by a difference equation using the relations between total, incident and reflected waves the recursive formula

$$\begin{aligned} {}_kV^i = {} & \frac{r + \kappa(1 - r^2) - 1}{r + \kappa(1 - r^2) + 1} \, {}_kV^r \\ & + \frac{\kappa(1 - r^2)}{r + \kappa(1 - r^2) + 1} ({}_{k-1}V^i - {}_{k-1}V^r) \end{aligned} \tag{18}$$

can be obtained.

Case 2: $r > 1$

Now the behavior of the connected line can approximately be described by a shunt connection of a resistor R_i and a capacitance C_i with

$$R_i \approx rZ_0 \tag{19}$$

and

$$C_i \approx \left(1 - \frac{1}{r^2}\right) \frac{l}{Z_0 c}. \tag{20}$$

Also replacing the differential equation

$${}_kI_{\text{tot}} = \frac{1}{R_i} \, {}_kV_{\text{tot}} + C_i \frac{d \, {}_kV_{\text{tot}}}{dt} \tag{21}$$

by a difference equation using once again the relations between total, incident and reflected waves the evaluation gives the recursive formula

$$\begin{aligned} {}_kV^i = {} & -\frac{\frac{1}{r} + \kappa\left(1 - \frac{1}{r^2}\right) - 1}{\frac{1}{r} + \kappa\left(1 - \frac{1}{r^2}\right) + 1} \, {}_kV^r \\ & + \frac{\kappa\left(1 - \frac{1}{r^2}\right)}{\frac{1}{r} + \kappa\left(1 - \frac{1}{r^2}\right) + 1} ({}_{k-1}V^i + {}_{k-1}V^r). \end{aligned} \tag{22}$$

Testing the recursion formulas in (18) and (22) for special cases should demonstrate their physical sensefulness in general. Therefore, the excess line is terminated by $Z_c = 0$ ($r = 0$, means electric wall) and $Z_c \to \infty$ ($r \to \infty$, means magnetic wall) leading to the same formulas as given in (5) using $\rho = -1$ and $\rho = 1$, respectively.

2.4 Time Varying Extensions

To model such phenomena like Doppler effect in time domain, it must be possible to move the boundaries during the simulation. A recursive formula for this shall now be derived on the example of the moving electric wall. Hence, the voltage reflection coefficient is -1, which can be represented by a short circuit. As shown above, this short circuit extension can be described by an inductance as long as we can assume that the extension is much smaller than the wavelength. For a time varying extension we have to distinguish between two cases once again. Therefore, it is necessary to know, if the varying in time is fast or slow.

In general, we have to solve the differential equation

$$V_{\text{tot}}(t) = \frac{\partial (L(t) I_{\text{tot}})}{\partial t} \tag{23}$$

in the time discrete TLM environment. We apply once again the identities for the total, incident and reflected values and replace the differential equation by a difference equation. The incident voltage at the time step k can then be calculated by

$$_kV^i = \frac{_k\kappa + (_k\kappa - {_{k-1}\kappa}) - 1}{_k\kappa + (_k\kappa - {_{k-1}\kappa}) + 1} + \frac{_k\kappa}{_k\kappa + (_k\kappa - {_{k-1}\kappa}) + 1}({_{k-1}V^i} - {_{k-1}V^r}). \tag{24}$$

Now, the new incident voltage depends on actual and previous incident and reflected voltages as well as on the actual and previous wall position, which has additionally to be stored for one time step. As long as the wall moves slowly in time (24) can be reduced to

$$_kV^i = \frac{_k\kappa - 1}{_k\kappa + 1} + \frac{_k\kappa}{_k\kappa + 1}({_{k-1}V^i} - {_{k-1}V^r}), \tag{25}$$

where only the actual wall position is needed for the evaluation.

III. Verification of Results

The accuracy of the above algorithm has been validated by performing extensive simulations of structures most sensitive to small variations in the dimensions, namely quarterwave and halfwave resonators as shown in Fig. 3. One of the walls was made moveable by application of (5), and 3-D TLM results obtained with the condensed node scheme [2] for the resonant frequencies were compared with accurate analytical values. Fig. 4 demonstrates the results. Data obtained for higher order modes yield information on the accuracy of the algorithm as a function of the angle of incidence. It appears that the error margin is largest for angles of incidence around 45 degrees.

The recursive formulas for arbitrary reflecting walls can be proved by testing them for some special cases. It can be shown that they yield the same equations as shown for simple reflection walls.

Since the positioning parameter κ can be changed after each computational step by an arbitrary small amount, it becomes feasible to model effectively boundaries that move at arbitrary speed during a simulation. For example, if κ is programmed to increase linearly in time, the boundary appears to be moving at a constant speed away from its initial position. This allows us to model directly the Doppler shift in the time domain. Fig. 5 shows the effect of wall movement on the shape and delay of a Gaussian impulse as modeled with 2D TLM. Two identical impulses of amplitude $+1$ have been reflected by electric walls (voltage reflection coefficient -1) and are propagating towards the left at velocity c. One of the reflecting walls was stationary, and the other started to move away from the source at constant speed $v = 0.035c$ at the moment of incidence. The delay, the widening and the amplitude reduction due to the wall movement are clearly noticeable. The TLM results correspond exactly to theoretical predictions and other numerical simulations [3].

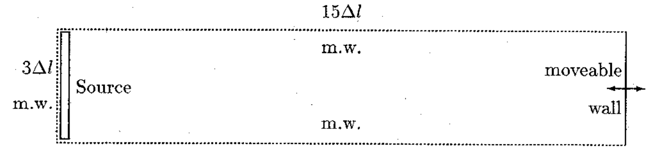

Fig. 3. Quarterwave resonator with movable sidewall for validation of the proposed algorithm.

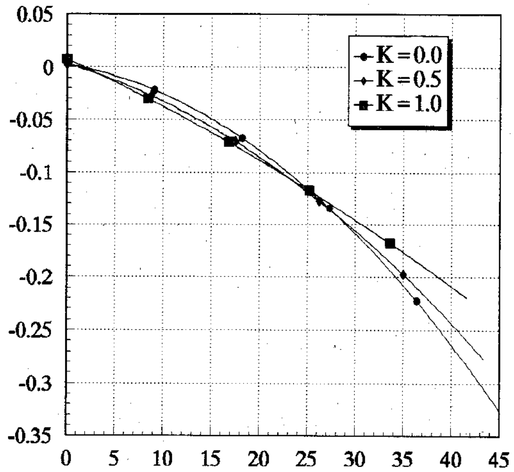

Fig. 4. Relative errors in resonant frequencies of the resonator shown in Fig. 3 as a function of the angle of incidence for three different values of the positioning parameter κ.

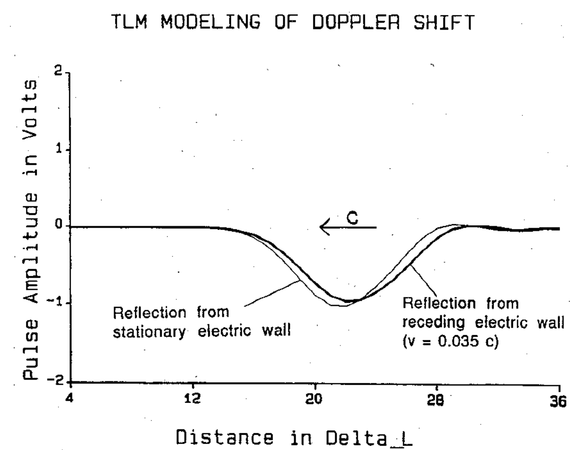

Fig. 5. The influence of wall movement on the shape and delay of a Gaussian impulse as modeled with 2-D TLM is shown. Two identical impulses of amplitude $+1$ have been reflected by electric walls and are propagating towards the left at velocity c.

IV. Conclusion

The new technique presented in this paper removes effectively the restriction that dimensions of TLM models can only be integer multiples of the mesh parameter. It thus considerably improves the flexibility of TLM modeling of microwave/millimeter-wave/optical components by freeing the modeler from the "Manhattan-style" approximation of curved boundaries and by improving the geometrical resolution without increase the computational expenditure. Since the position parameter κ ($\kappa = \sqrt{2}l/\Delta l$ for 2D TLM and $\kappa = 2l/\Delta l$ for 3-D SCN TLM) can varied in arbitrarily small increments between computational steps, this feature can be used to model moving boundaries and allows optimization in the time domain by modification of structure geometry during a simulation. Also, the direct visualization of phenomena such as the Doppler effect becomes feasible.

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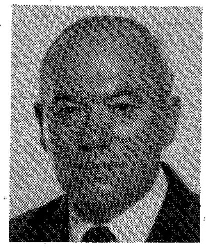

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